

MENSURATION

(P1 + P2)(20 MARKS)

FORMULAS

UNIT
CONVERSIONS

SIMILARITY

2-D

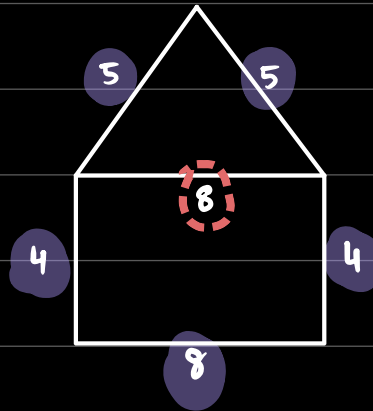
AREA

↓
Memorize the formula.

PERIMETER

↓
DO NOT MEMORIZE ANY
FORMULA FOR PERIMETER.

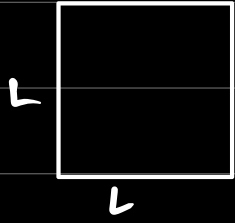
Def. SUM OF ALL OUTER SIDES



$$\text{Perimeter} = 5 + 5 + 4 + 4 + 8 = 26$$

1 SQUARE

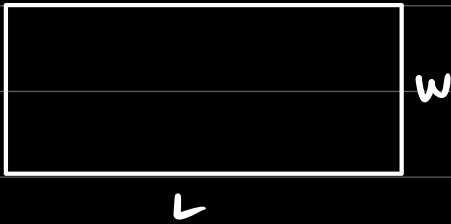
$$\text{Area} = L \times L = L^2$$



Perimeter: Do not memorize this formula. This is just for practice.

$$P = L + L + L + L = 4L$$

2

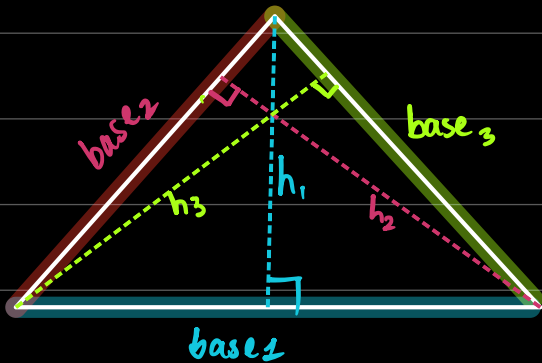


$$\text{Area} = L \times w$$

$$P = 2L + 2w = 2(L + w)$$

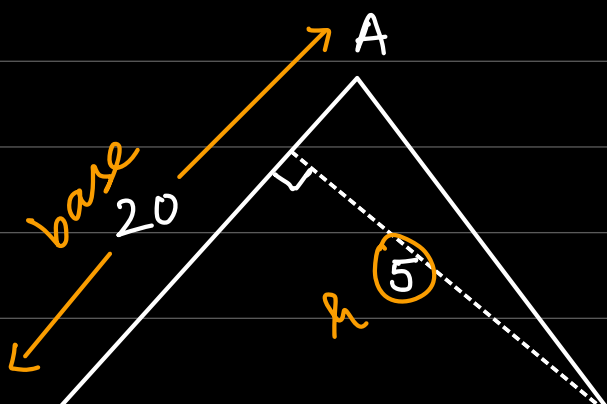
3

AREA OF TRIANGLE



$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$h = 90^\circ$ distance from THIRD point on base line.



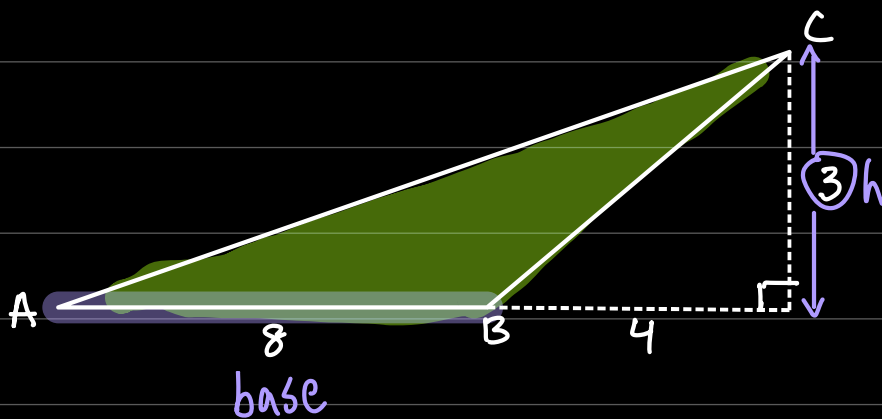
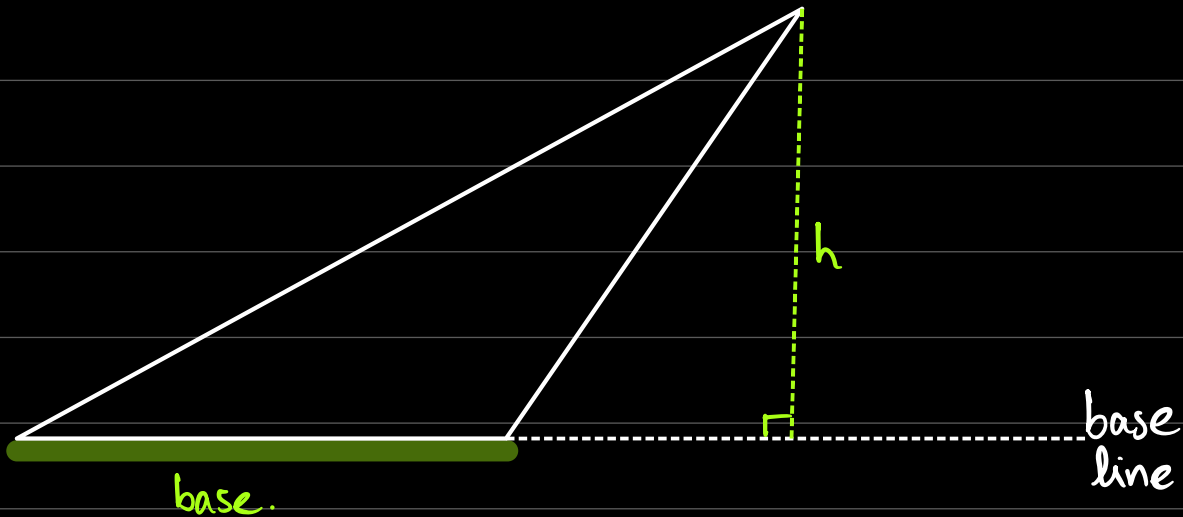
Find area of $\triangle ABC$.

$$A = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 20 \times 5 = 50$$

B $\xrightarrow{12}$ C 1

SPECIAL TRIANGLE TO REMEMBER



Find area of Triangle ABC.

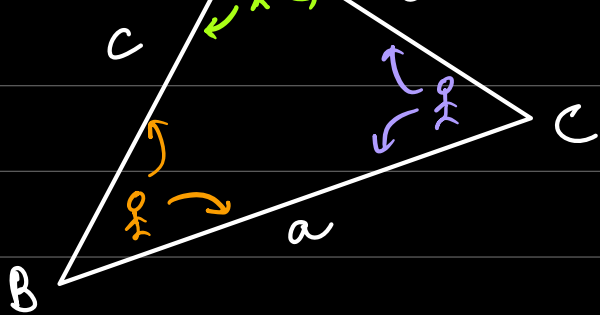
$$A = \frac{1}{2} \times b \times h$$

$$A = \frac{1}{2} \times 8 \times 3 = 12$$

IF NO PAIR OF CORRECT BASE AND HEIGHT IS GIVEN:



$$A = \frac{1}{2} \square \square \sin \bigcirc$$



2 $\overbrace{\hspace{2cm}}$
neighboring
sides of
that angle.

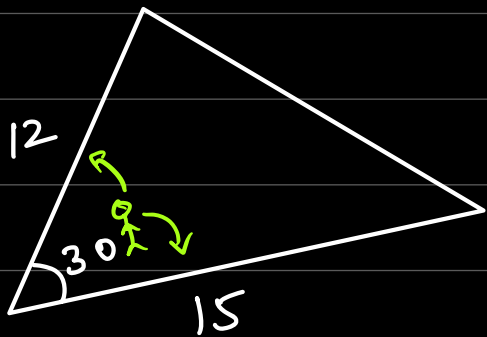
Angle

$$A = \frac{1}{2} \boxed{c} \boxed{b} \sin(\textcircled{A})$$

$$A = \frac{1}{2} \boxed{a} \boxed{c} \sin(\textcircled{B})$$

$$A = \frac{1}{2} \boxed{a} \boxed{b} \sin(\textcircled{C})$$

Q.

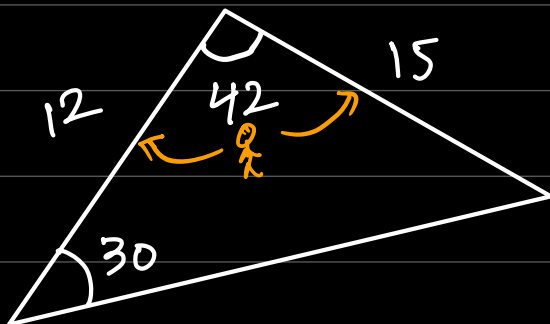


find area of Triangle.

$$A = \frac{1}{2} \boxed{12} \boxed{15} \sin(\textcircled{30})$$

$$= 45$$

Q.



find area of triangle.

$$A = \frac{1}{2} \boxed{12} \boxed{15} \sin(\textcircled{42})$$

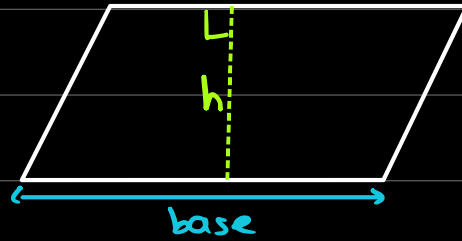
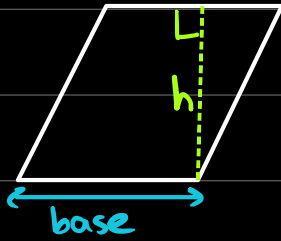
$$= 60.22$$

4

RHOMBUS

Σ

PARALLELOGRAM

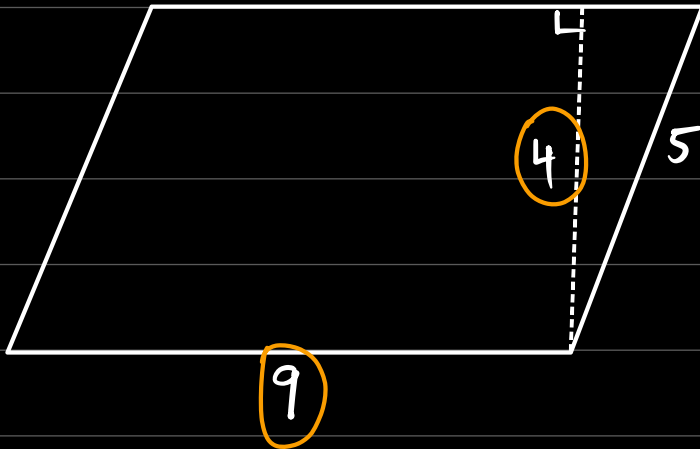


$$\text{Area} = \text{Base} \times h$$



$h = 90^\circ$ distance between parallel lines.

Q

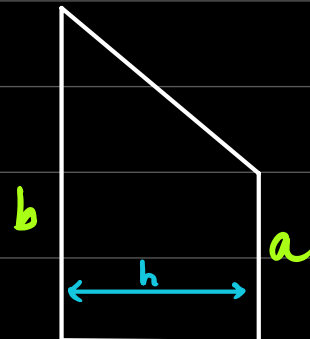
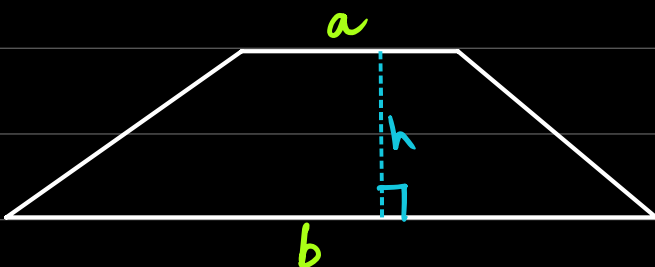


Find area of parallelogram.

$$\begin{aligned} \text{Area} &= b \times h \\ &= 9 \times 4 \\ &= 36 \end{aligned}$$

5

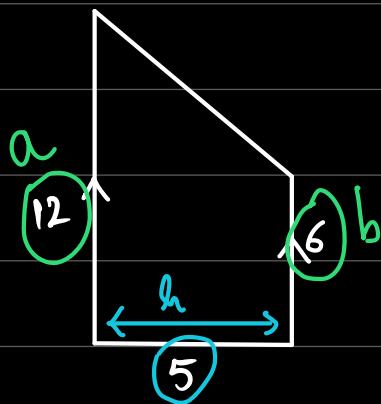
TRAPEZIUM



$$A = \frac{1}{2} \times h \times (\text{sum of parallel sides})$$

$$A = \frac{1}{2} \times h \times (a+b)$$

$h = 90^\circ$ distance between parallel sides.

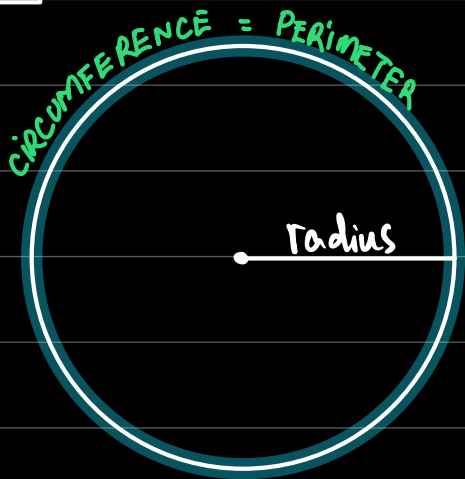


$$\text{Area} = \frac{1}{2} \times h \times (a+b)$$

$$A = \frac{1}{2} \times 5 \times (12+6)$$

$$A = \frac{1}{2} \times 5 \times 18 = 45$$

6 CIRCLE



$$\text{Area} = \pi r^2$$

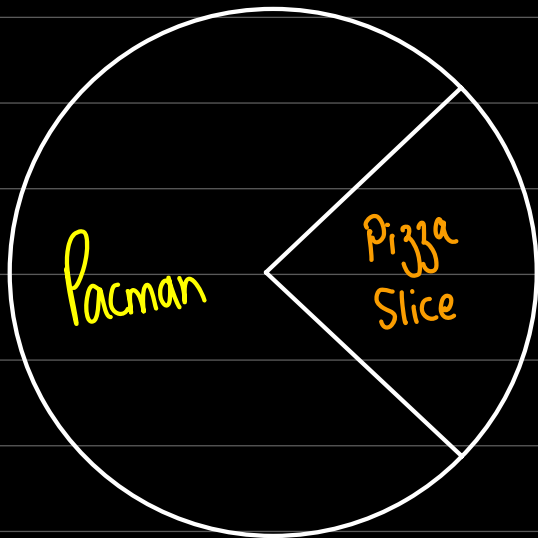
$$\text{Perimeter} = 2\pi r$$

↓
Circumference

P2) Use π on calculator: SHIFT MIDDLE BUTTON ON BOTTOM LINE

P1) Question will tell us value of π

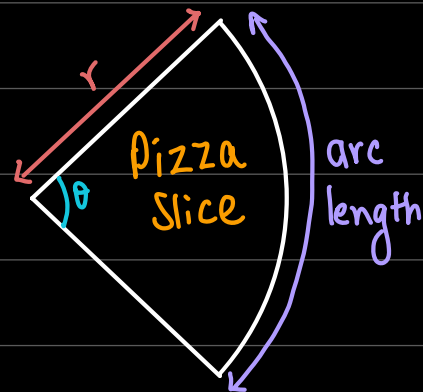
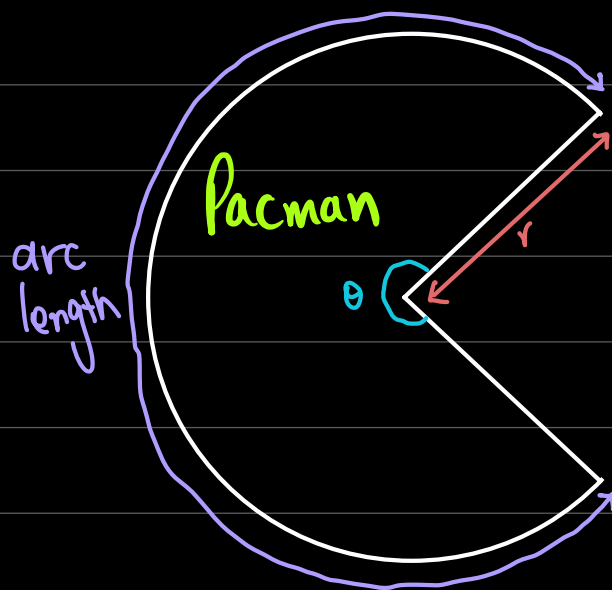
If question is silent, keep writing π until it gets cancelled in the end.



SECTORS



SEGMENTS



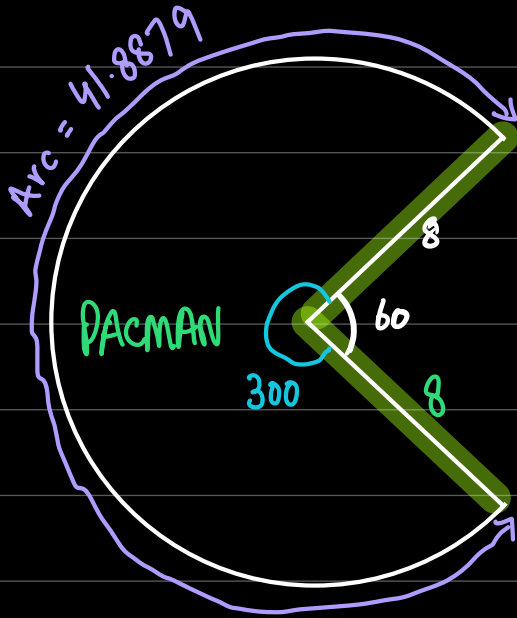
SECTORS

Pacman and Pizza slice are good friends. They use the exact same formulas.

$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r$$

$$\text{Area of Sector} = \frac{\theta}{360} \times \pi r^2$$

Q:



(i) Find area of sector.

$$\text{Area} = \frac{\theta}{360} \times \pi r^2 = \frac{300}{360} \times \pi (8)^2 = 167.5516$$

(ii) Find arc length

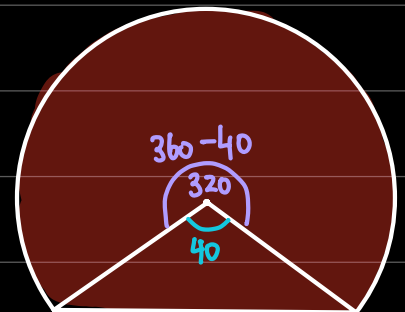
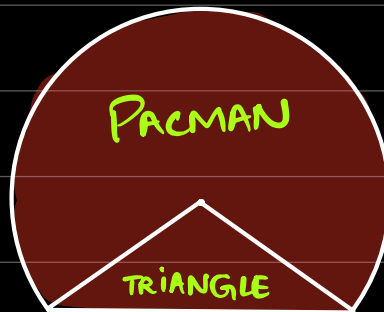
$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r = \frac{300}{360} \times 2\pi (8) = 41.8879$$

(iii) Find perimeter of Sector

$$P = 8 + 8 + 41.8879 \\ = 57.8879$$

SEGMENTS

Area of major segment

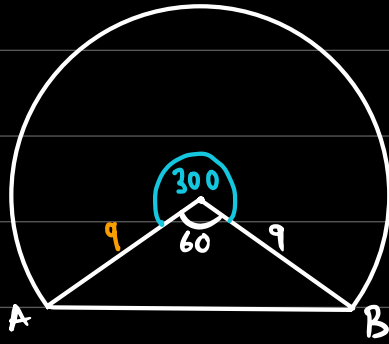


Area of major Segment = PACMAN + TRIANGLE

$$\left[\frac{\theta}{360} \times \pi r^2 \right] + \left[\frac{1}{2} \square \square \sin \bigcirc \right]$$

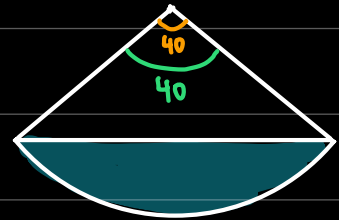
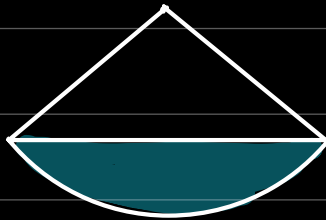
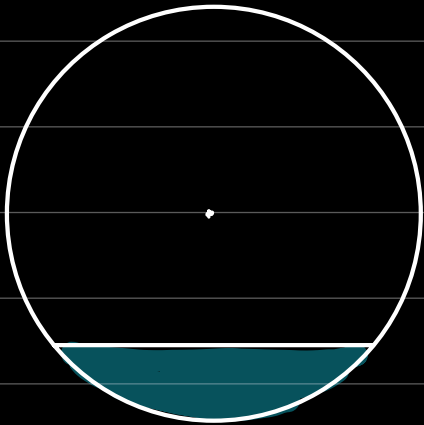
NOTE: IN MAJOR SEGMENT, PACMAN AND TRIANGLE BOTH USE TWO DIFFERENT ANGLES

(2012) FIND THE AREA OF MAJOR SEGMENT (4 MARKS)



$$\begin{aligned}
 \text{MAJOR SEGMENT} &= \text{PACMAN} + \text{TRIANGLE} \\
 &= \left[\frac{\theta}{360} \times \pi r^2 \right] + \left[\frac{1}{2} \square \square \sin \theta \right] \\
 &= \left[\frac{300}{360} \times \pi (9)^2 \right] + \left[\frac{1}{2} \square \square \sin 60 \right] \\
 &= 212.0575 + 35.0740 \\
 &= 247.1315
 \end{aligned}$$

AREA OF MINOR SEGMENT

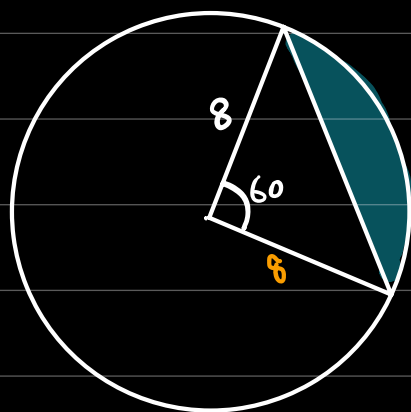


MINOR SEGMENT = PIZZA SLICE - TRIANGLE

$$\left[\frac{\theta}{360} \times \pi r^2 \right] - \left[\frac{1}{2} \square \square \sin \theta \right]$$

NOTE: FOR MINOR SEGMENT, BOTH TRIANGLE AND PIZZA SLICE USE SAME ANGLE.

Find area of shaded region



$$\begin{aligned}
 \text{MINOR SEGMENT} &= \text{PIZZA} - \text{TRIANGLE} \\
 &= \left[\frac{\theta}{360} \times \pi r^2 \right] - \left[\frac{1}{2} \square \square \sin \bigcirc \right] \\
 &= \left[\frac{60}{360} \times \pi (8)^2 \right] - \left[\frac{1}{2} [8][8] \sin (60) \right] \\
 &= 33.5103 - 27.7128 \\
 &= 5.7975
 \end{aligned}$$

3-D SHAPES

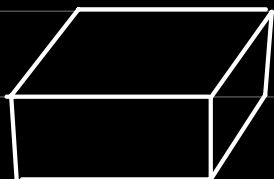
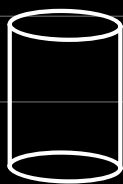
VOLUME

Memorize
Formula

EXTERNAL SURFACE AREA

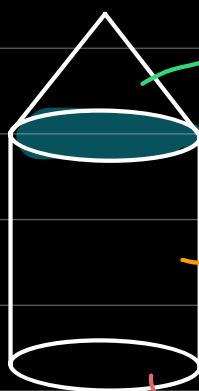
Never memorize formula.

Def: Sum of areas of all
outer faces of a shape.



open containers

Top surface is missing.



③ Curved Surface
area of a cone.

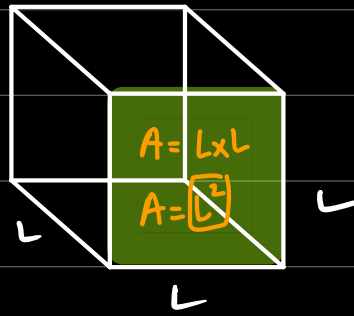
② Curved Surface
area of a
cylinder

① Circle = πr^2

1 CUBE

$$V = L \times L \times L = L^3$$

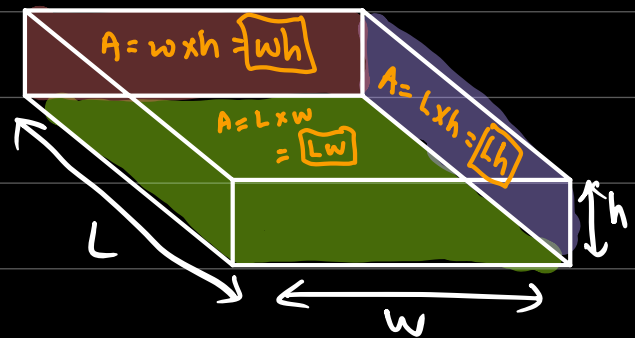
$$TSA = 6 \times L^2 = 6L^2$$



2 CUBOID

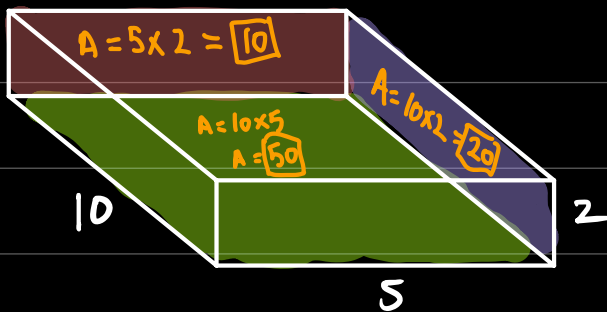
$$V = L \times w \times h$$

$$TSA = 2(Lw) + 2(Lh) + 2(wh)$$



Top surface is missing

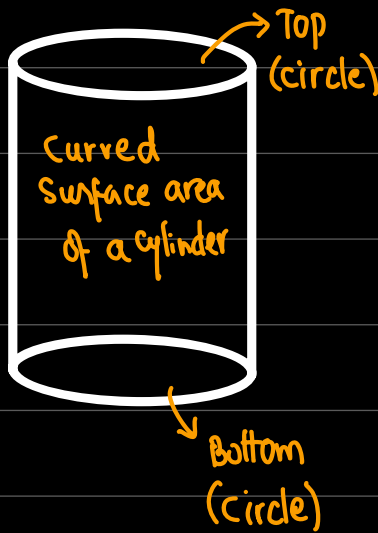
Q: An open cuboid container is shown.
Find total external surface area?



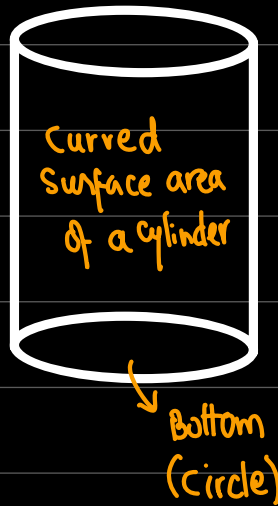
$$\begin{aligned} \text{Total SA} &= 50 + 2(20) + 2(10) \\ &= 50 + 40 + 20 \\ &= 110 \end{aligned}$$

CYLINDER

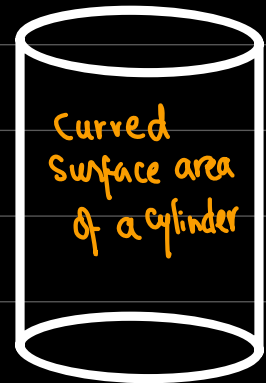
CLOSED TIN
(PEPSI CAN)



OPEN CONTAINER
(GLASS)

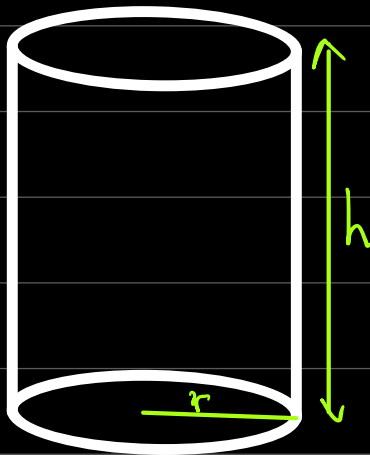


PIPE



1) VOLUME IS SAME FOR ALL THREE TYPES.

$$\text{VOLUME} = \pi r^2 h$$



TOTAL SURFACE AREA

1) TOP = CIRCLE = πr^2

2) BOTTOM = CIRCLE = πr^2

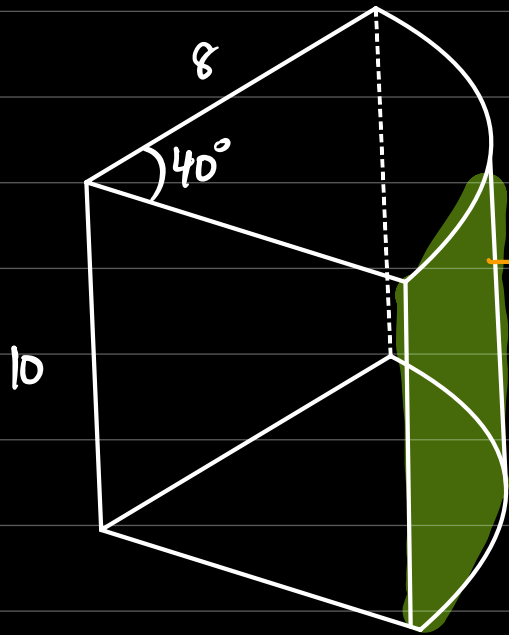
3) Curved Surface area of cylinder

$$\text{CSA of Cylinder} = \left[\text{Arc length} \right] \times h$$

$$\left[\frac{\theta}{360} \times 2\pi r \right] \times h$$

For a full cylinder = $\theta = 360$ $\rightarrow \left[\frac{360}{360} \times 2\pi r \right] \times h = 2\pi r h \rightarrow \text{Book}$

Q. Find Curved Surface area of this cake slice.



Curved SA of cylinder = Arc length $\times h$

$$= \left[\frac{\theta}{360} \times 2\pi r \right] \times h$$

$$= \left[\frac{40}{360} \times 2\pi(8) \right] \times 10$$

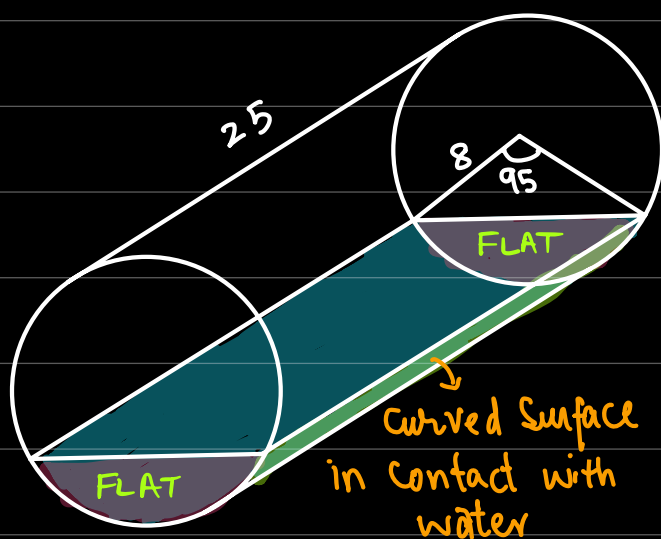
$$= 55.8505$$



Since cake is a cylinder,

we use same formulas for curved SA of cake slice too.

2



Find the curved surface area of Container in contact with water.

$$\text{Curved SA} = \text{Arc of cylinder} \times \text{length}$$

$$= \left[\frac{\theta}{360} \times 2\pi r \right] \times h$$

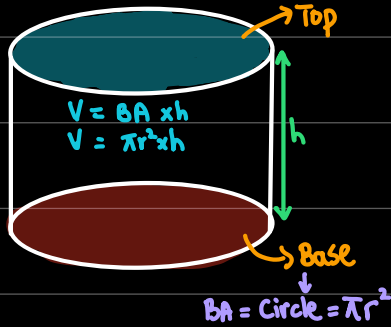
$$= \left[\frac{95}{360} \times 2\pi(8) \right] \times 25$$

$$= 331.6125$$

UPRIGHT SHAPES

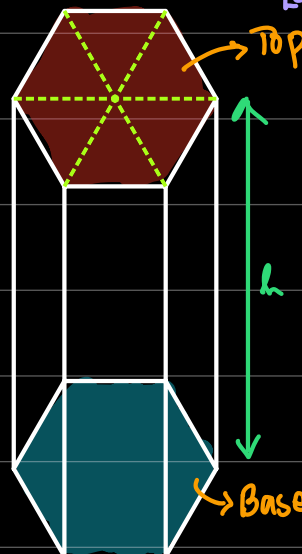
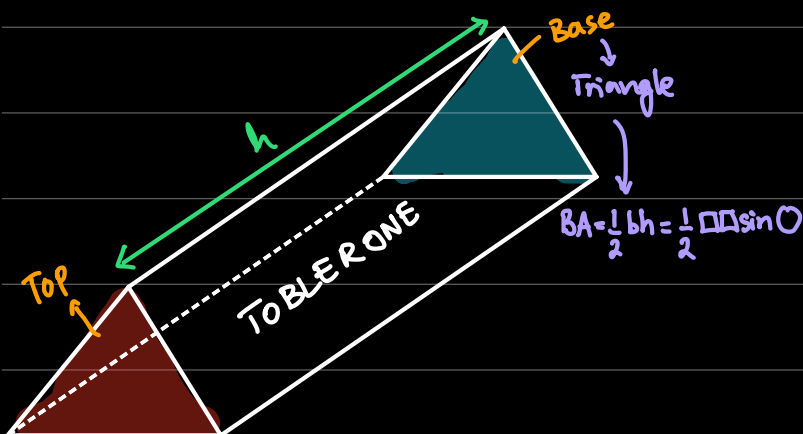
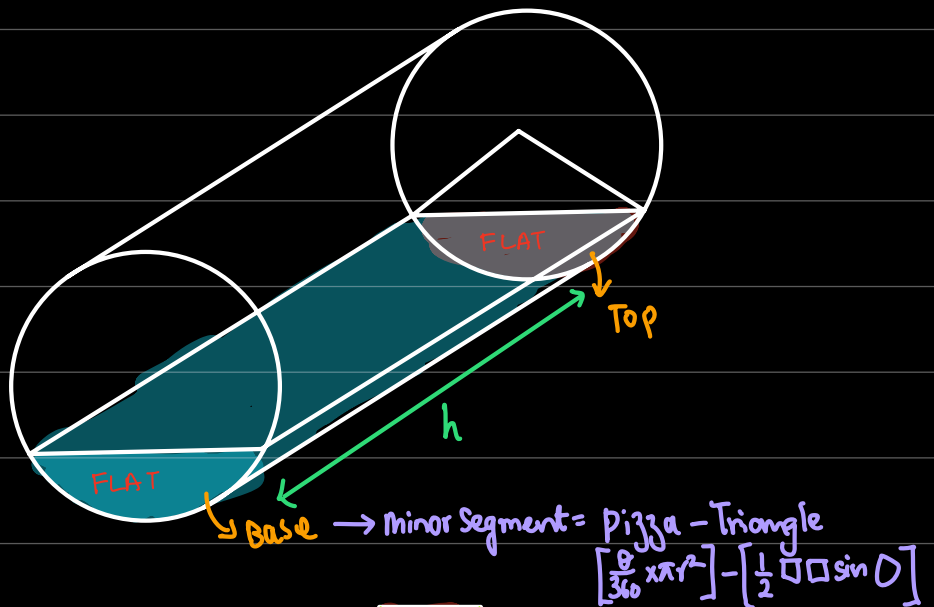
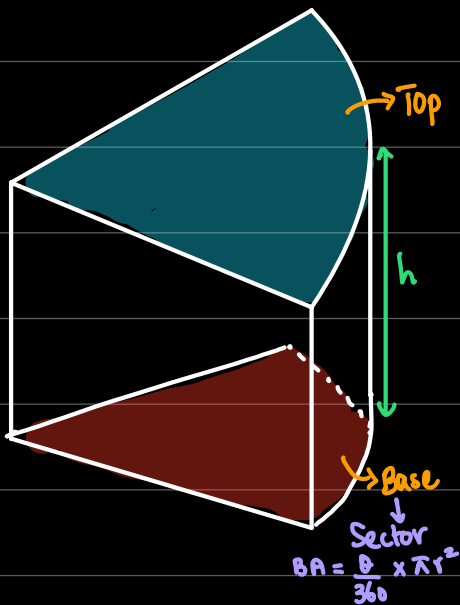
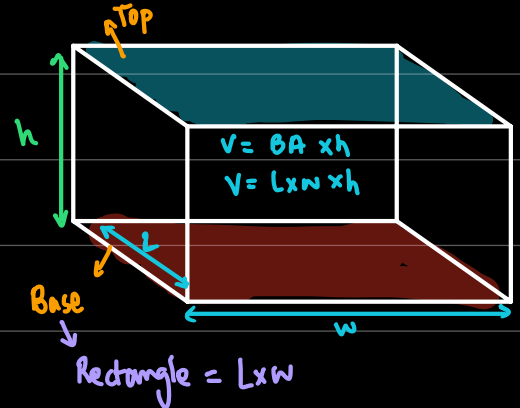
PRISMS

TWO EQUAL AND PARALLEL FACES.



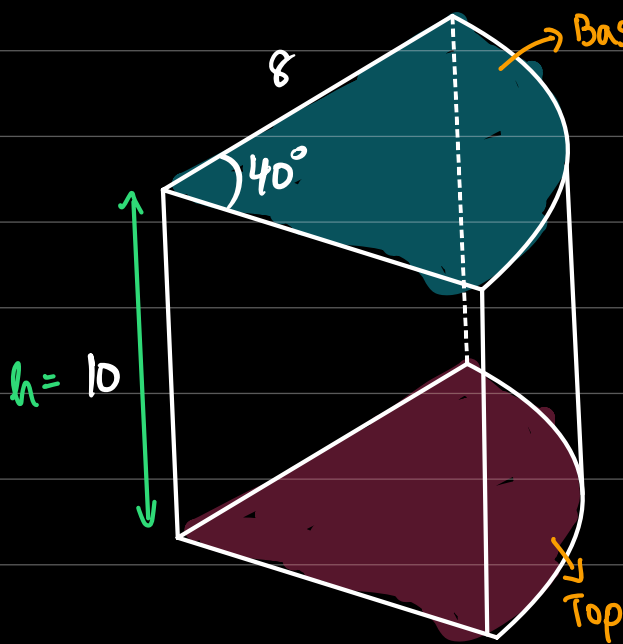
$$\text{VOLUME} = \text{Base} \times h$$

Base = Area of cross-section



Question will always help us to find this area.

Find volume of this cake slice.



$$\text{Volume} = \text{Base area} \times h$$

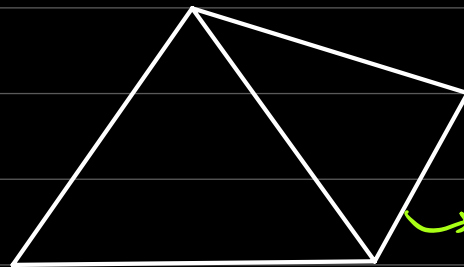
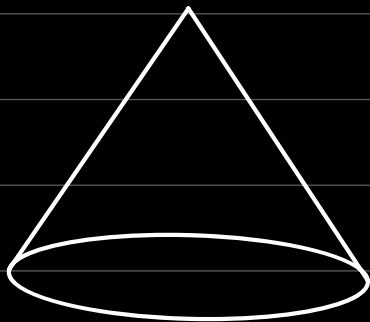
Sector
(Pizza)

$$= \left[\frac{\theta}{360} \times \pi r^2 \right] \times h$$

$$= \left[\frac{40}{360} \times \pi (8)^2 \right] \times 10$$

$$V = 223.4021$$

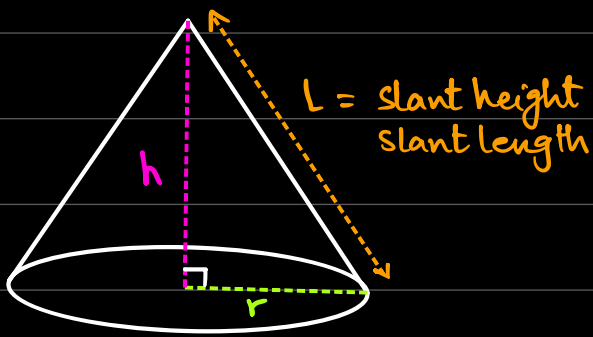
POINTED SHAPES



Base of a
Pyramid can
be any shape.

$$V = \frac{1}{3} \times \text{Base Area} \times h$$

CONE



1) Pythagoras

$$L^2 = h^2 + r^2$$

2) VOLUME

$$V = \frac{1}{3} \times \underbrace{\text{Base area}}_{\text{Circle} = \pi r^2} \times h$$

$$V = \frac{1}{3} \times \pi r^2 \times h$$

3 TOTAL SURFACE AREA :

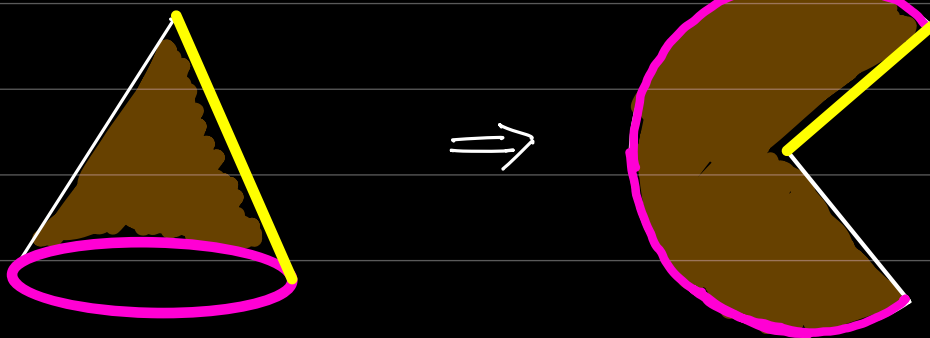
$$\text{BASE} = \text{CIRCLE} = \pi r^2$$

$$\text{Curved Surface area of cone} = \pi r l$$

WHAT HAPPENS WHEN A CONE IS OPENED???

It becomes a SECTOR

can be pizza slice or Pacman

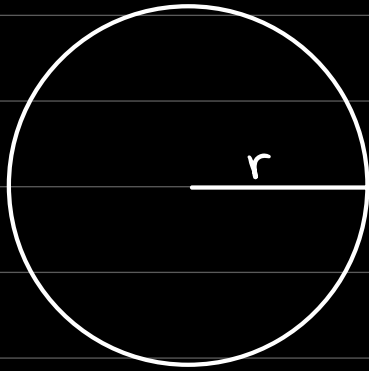


1-) SLANT HEIGHT OF CONE $\xrightarrow{\text{BECOMES}}$ RADIUS OF SECTOR

2-) CIRCUMFERENCE OF CONE BASE $\xrightarrow{\text{BECOMES}}$ ARC LENGTH OF SECTOR

3-) CURVED SA OF CONE $\xrightarrow{\text{BECOMES}}$ AREA OF SECTOR.

SPHERES



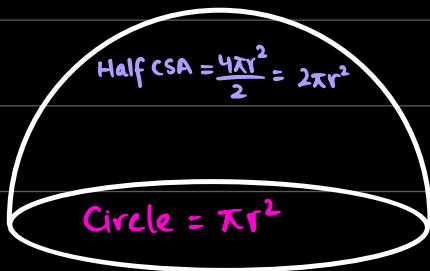
$$\text{VOLUME} = \frac{4}{3} \pi r^3$$

$$\text{Curved Surface area of sphere} = 4\pi r^2$$

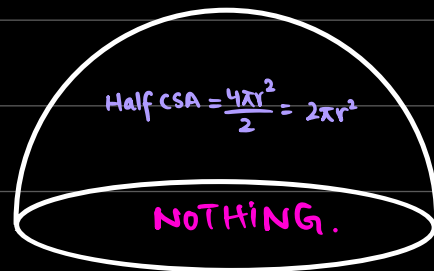
HEMISPHERE

DO NOT MEMORIZE ANY NEW FORMULAS FOR HEMISPHERES.

SOLID



HOLLOW.



BOTH HAVE SAME VOLUME:

$$V = \frac{4}{3} \pi r^3 \div 2$$

CURVED SURFACE AREA

CYLINDER

$$\text{CSA} = \text{Arc length} \times h$$

For full cylinder, $\theta = 360$

CONE

$$\text{CSA} = \pi r l$$

SPHERE

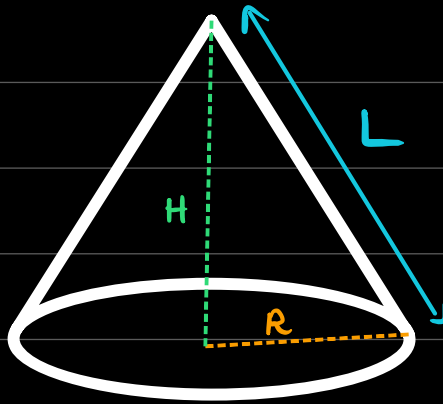
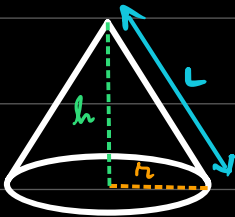
$$\text{CSA} = 4\pi r^2$$

SIMILARITY

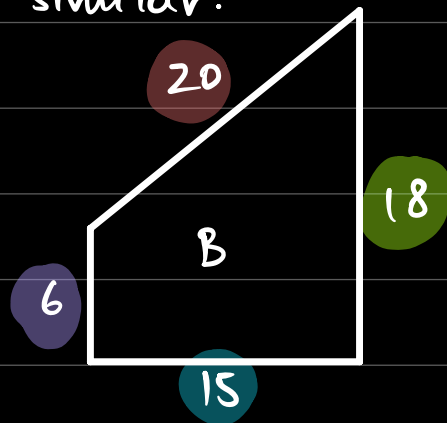
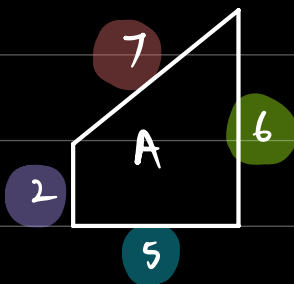
1) AAA = If all angles of two different sized shapes are equal, they are SIMILAR.

2) FOR SIMILAR SHAPES, RATIO OF CORRESPONDING LENGTHS IS EQUAL
Divide

$$\frac{h}{H} = \frac{L}{L} = \frac{r}{R}$$



Q: Check if both shapes are similar.



$$\frac{B}{A} = \frac{6}{2} = \frac{15}{5} = \frac{18}{6} = \frac{20}{7}$$

3 3 3 Not 3

A and B are not similar.

IF TWO SHAPES ARE **SIMILAR**,
WE DO NOT APPLY MEASUREMENT FORMULAS
TO FIND AREA & VOLUME.

AREA

$$\frac{A_1}{A_2} = \left(\frac{L_1}{L_2} \right)^2$$

$\frac{L_1}{L_2}$ = ratio of
corresponding
sides.

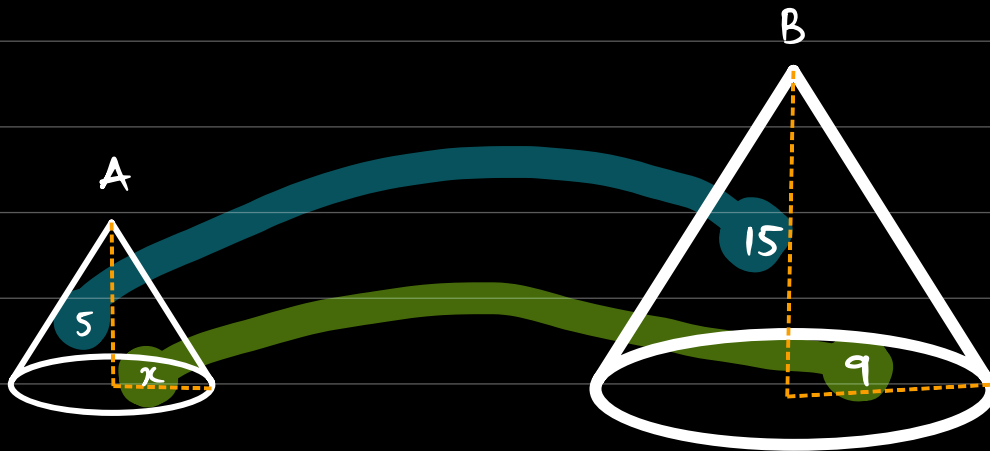
VOLUME

$$\frac{V_1}{V_2} = \left(\frac{L_1}{L_2} \right)^3$$

Mass

$$\frac{M_1}{M_2} = \left(\frac{L_1}{L_2} \right)^3$$

Q. Cone A and Cone B are **Similar**.



$$\frac{x}{9} = \frac{5}{15}$$

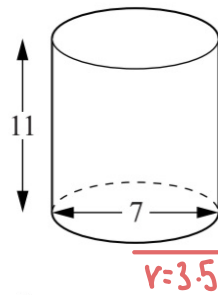
$$3x = 9$$

$$x = 3$$

Answer [2]

- (b) A closed cylindrical tin is 11 cm high and the base has a diameter of 7 cm.

Three surfaces



- (i) Calculate the volume of this tin.

$$V = \pi r^2 h = \pi (3.5)^2 (11)$$
$$V = 423.3296$$

Answer cm^3 [2]

(ii) Calculate the total external surface area of this tin.

$$\begin{aligned}
 1) \text{ Top} &= \text{circle} = \pi r^2 = \pi (3.5)^2 = 38.4845 \\
 2) \text{ Base} &= \text{circle} = 38.4845 \\
 3) \text{ Curved SA of cylinder} &= \left[\frac{\text{Arc}}{\text{length}} \times h \right] = \left[\frac{\theta}{360} \times 2\pi r \right] \times h = \left[\frac{360}{360} \times 2\pi (3.5) \right] \times 11 = 241.9026 \\
 \text{Total Surface area} &= 318.8716
 \end{aligned}$$

Answer 318.8716 cm² [3]

(iii) In addition to the surface area, a closed tin requires an extra 30 cm² of metal to allow the top, bottom and side to be joined together.

Calculate the area of metal required for 30 000 closed tins.
Give your answer in square metres

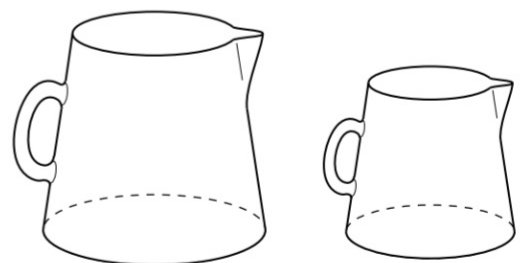
cm² $\xrightarrow{\text{divide by 100}}$ m²

$$\begin{aligned}
 1 \text{ tin} &= 318.8716 + 30 = 348.8716 \text{ cm}^2 \\
 &\quad \times 30000 \text{ tins} \\
 \hline
 &= \frac{10466148 \text{ cm}^2}{(100)(100)} = 1046.6148 \text{ m}^2
 \end{aligned}$$

Answer 1046.6148 m² [2]

(c) Two geometrically similar jugs have volumes of 1000 cm³ and 512 cm³. They have circular bases. The diameter of the base of the larger jug is 9 cm.

Calculate the diameter of the base of the smaller jug.



$$\begin{aligned}
 \frac{V_1}{V_2} &= \left(\frac{L_1}{L_2} \right)^3 & \frac{1000}{512} &= \frac{729}{d^3} \\
 \frac{1000}{512} &= \left(\frac{9}{d} \right)^3 & 1000d^3 &= 729 \times 512 \\
 & & d^3 &= \frac{729 \times 512}{1000} \\
 & & d^3 &= 373.248 \\
 & & d &= \sqrt[3]{373.248} \\
 & & d &= 7.2
 \end{aligned}$$

Answer 7.2 cm [3]